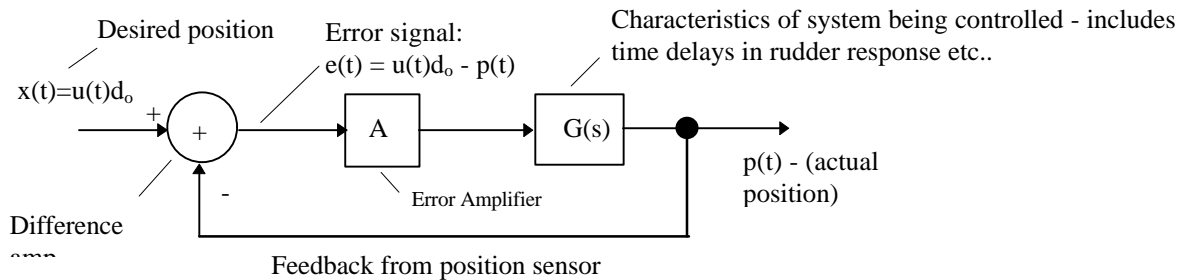


Musings on Feedback Control Systems
- by D.J. Mehrl, 11/24/98 -

Typical feedback system for tracking a wall:

- 1.) Assume a distance sensor outputs an analog voltage $v(t)$ which is proportional to the distance from the wall.
- 2.) In response to the signal, the rudders are turned at an appropriate angle so as to keep the boat tracking at some desired distance d_o , where v_{ref} is the analog output of the distance sensor corresponding to this distance d_o .
- 3.) Let $G(s)$ represent the dynamics of the boat's response. That is, when you turn the rudder, the boat will not immediately go to the desired distance d_o . It will take some time to get there. A typical feedback control system



looks like:

This simple feedback system is known as a "proportional" control system, which I assume is what will be used by Project Lab I students. Better (but more complicated) control systems include the PI controller (proportional/integral feedback controller) and the PID controller (proportional/integral/derivative) feedback controller.

In your application, the desired distance from the wall, d_o , will likely be fixed (e.g. at 6"). $G(s)$ is a Laplace domain representation of the system dynamics, and we will formulate an approximate model for it later. For now, let's assume that there is a constant time delay of t_o , represented by the impulse response $g(t) = \delta(t - t_o)$. Then, $G(s) = \exp(-st_o)$. Now, let $p(t)$ have the Laplace transform $P(s)$, i.e. $p(t) \leftrightarrow P(s)$, $e(t) \leftrightarrow E(s)$, and $e(t) \leftrightarrow E(s)$. We also see that $u(t)d_o \leftrightarrow d_o/s$.

In general, we find that $P(s) = A G(s) [X(s) - P(s)]$, or rearranging, $P(s) [1 + A G(s)] = A G(s) X(s)$,

It is interesting to derive the transfer function $T(s) = \frac{P(s)}{X(s)} = \frac{A G(s)}{1 + A G(s)}$.

Now, $P(s) = X(s)T(s)$. If $T(s) = 1$, then $P(s) = X(s)$, or the actual position is always exactly equal to the desired position. Note that this can be realized by making A , the (constant) error amplifier gain, very large, as:

$$\lim_{A \rightarrow \infty} \frac{A G(s)}{1 + A G(s)} = 1.$$

Unfortunately, a "gotcha" exists! If the denominator of the above expression = 0, then $|T(s)| \rightarrow \infty$, i.e. the system becomes unstable (i.e. it will oscillate). This oscillation can occur because of the existence of the closed feedback loop in the above control system! When the closed loop gain = 1, i.e. $-A G(s) = 1$, the condition required for oscillation is met. In this case, we have $\exp(-st_o) = -1/A$, or $\exp[-(\sigma + j\omega)t_o] = \exp[-\sigma t_o] \exp[-j\omega t_o] = -1/A$. This condition can be met if: $\omega = m\pi/t_o$, where m is any odd integer [this will result in $\exp(-j\omega t_o) = -1$], and if: $\sigma = (1/t_o) \ln A$ [this will result in $\exp(-\sigma t_o) = 1/A$]. What are the chances of s assuming this particular value, you ask? If the above system can oscillate, it will always find, i.e. actually seek out, a way to do so!!!! The chances of

the system oscillating generally increases when A is made larger! This is very similar to the feedback oscillation incurred in an auditorium when the rock band turns up their PA amplifier too high! The system finds certain frequencies that are just right where the closed loop gain is exactly equal to unity, and the PA system produces a loud obnoxious tone at that frequency! Hence there is a tradeoff between making the system accurate and responsive (i.e. making $T(s)$ approach unity) , and keeping the system stable!

Because most project lab I students haven't had any exposure to control theory, we will not dwell further on it, other than to recommend a general strategy for a proportional controller. By taking these steps, you can avoid hardware, and let the 68HC1x do all the work while offering great flexibility. E.g., you can change the control system by changing 68HC1x code and/or adjusting some external potentiometers.

(I.) Construct the above system, allowing for variable gain (A might be adjusted, e.g., by interfacing a potentiometer based voltage divider to one of the 68HC1x A-to-D inputs). It might be reasonable to, say, set things up so that A can be varied between the range $1 < A < 30$ by adjusting an external potentiometer.

(II.) Allow for a potentiometer controlled "desired position", again by interfacing a potentiometer based voltage divider to one of the 68HC1x A-to-D inputs. This way, you can quickly change things during competition time (e.g. if you need to get your boat to track further away from the wall), by merely tweaking a potentiometer.

(III.) Don't actually build an op-amp based difference circuit. Let the feedback (measured position) come into the 68HC1x, and subtract that from the desired position using 68HC1x code.

(IV.) Of course, you've set things up so that the 68HC1x can control the steering, thus $P(s)$ is somehow controlled by the 68HC1x output ports.

(V.) Now play around with the gain (A) until you get reasonable performance.

Dynamics of steering:

To gain some appreciation of system dynamics, let's try to crudely model a boat's steering. Firstly, the boat has some moment of inertia, J, such that it will not immediately respond to a rudder direction change. The lateral force on the rudder will be a function of the rudder angle, ϕ , as well as the forward velocity (V) of the boat - let's assume that the "angular acceleration" α is given by, $\alpha(t) = \beta V \phi(t)/J$ where β is some undetermined proportionality constant which depends, e.g., on the area of the rudder, and J is the "moment of inertia" of the boat hull. Now the

rotational speed, Ω , is found as $\Omega(t) = \beta V/J \int_0^t \phi(\tau) d\tau$, and $\theta(t) = \int_0^t \Omega(\tau) d\tau$ etc..

Secondly, assume the boat is moving at a constant forward velocity \mathbf{V} , but is not at the desired distance from the wall. Note that \mathbf{V} is a vector quantity, but the velocity component in the direction normal to the wall would likely

be given as $V_x(t) = V \sin \theta(t)$. Now the position with respect to the wall is found as $p_x(t) = \int_0^t V \sin \theta(\tau) d\tau$ Well

then it will take time for the boat to move from it's actual position to the desired position. We could work out a simple model of the boat dynamics, but it would probably be beyond the scope of what students can understand. We just want to give the student an appreciation for some of the complexities and considerations involved.

